Lecture Notes on **Discrete Mathematics**.
Birzeit University, Palestine, 2015

First Order Logic

Mustafa Jarrar

3.1 Introduction to First Order Logic

3.2 Negation and Conditional Statements

3.3 Multiple Quantifiers in First Order Logic

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Watch this lecture and download the slides

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Acknowledgement:
This lecture is based on (but not limited to) to chapter 3 in "Discrete Mathematics with Applications by Susanna S. Epp (3rd Edition)".

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First Order Logic

3.3 Multiple Quantifiers

In this lecture:

- Part 1: **Multiple and Order of Quantifiers**
- Part 2: Formalize/Verbalize Multiple Quantifiers
- Part 3: Negations of Multiply-Quantified Statements
- Part 4: Example: Using FOL to formalize text (optional)

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Multiple and Order of Quantifiers

$\forall x\ \exists y\ .\ \text{Loves}(x,y)$ Everything loves something Each thing loves one or more things	$\exists x\ \forall y\ .\ \text{Loves}(x,y)$ Something loves everything
$\forall y\ \exists x\ .\ \text{Loves}(x,y)$ Everything is loved by something Everything have something that loves it	$\forall y\ \exists x\ .\ \text{Loves}(y,x)$ Everything loves something
$\exists y\ \forall x.\ \text{Loves}(x,y)$ Something is loved by everything Everything love the same thing There exists something that everything loves it	$\forall x\ \exists y\ .\ \text{Loves}(x,y),\ x \neq y$ Everything loves something but not itself
$\forall x\ \forall y\ .\ \text{Loves}(x,y)$ $\forall x,y\ \ .\ \text{Loves}(x,y)$ Everything loves everything	$\exists x\ \exists y\ .\ \text{Loves}(x,y)$ $\exists x,y\ \ .\ \text{Loves}(x,y)$ something loves something

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Multiple and Order of Quantifiers

Everyone loves all movies
كل شخص يحب كل الافلام

$\forall p_{\in \text{Person}}\ \forall m_{\in \text{Movie}} \cdot \text{Loves}(p,m)$

Some people loves some movies
بعض الناس يحبون بعض الافلام

$\exists p_{\in \text{Person}}\ \exists m_{\in \text{Movie}} \cdot \text{Loves}(p,m)$

There is a movie that everyone loves
فلم يحبه كل الناس

$\exists m_{\in \text{Movie}}\ \forall p_{\in \text{Person}} \cdot \text{Lovedby}(m,p)$

Some people love all movies
بعض الناس يحب كل الافلام

$\exists p_{\in \text{Person}}\ \forall m_{\in \text{Movie}} \cdot \text{Loves}(p,m)$

Everyone loves some movies
كل شخص يحب بعض الافلام

$\forall p_{\in \text{Person}}\ \exists m_{\in \text{Movie}} \cdot \text{Loves}(p,m)$

All movies are loved by someone
كل فلم له بعض المحبين

$\forall m_{\in \text{Movie}}\ \exists p_{\in \text{Person}} \cdot \text{Lovedby}(m,p)$

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First Order Logic

3.3 Multiple Quantifiers

In this lecture:

- ❑ Part 1: Multiple and Order of Quantifiers
- ❑ Part 2: **Formalize/verbalize multiple Quantifiers** **(more examples)**
- ❑ Part 3: Negations of Multiply-Quantified Statements
- ❑ Part 4: Example: Using FOL to formalize text (optional)

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Multiple Quantifiers with Negated Predicates

$\exists x\ \exists y\ .\ \sim Love(x,y)$

Somebody does not love somebody

بعض اشخاص لا يحبون بعض الاشخاص

$\forall x\ \forall y\ .\ \sim Love(x,y)$

Everyone does not love anyone

No one love any one.

كل شخص لا يحب أي شخص

لا احد يحب احد

$\exists x\ \forall y\ .\ \sim Love(x,y)$

Someone does not love anyone

يوجد شخص لا يحب أي شخص

$\forall x\ \exists y\ .\ \sim Love(y,x)$

Everyone is not loved by someone

Everyone has some people who do not love him

لكل شخص يوجد اخرين لا يحبونه

* something/somebody ,everyone/everything

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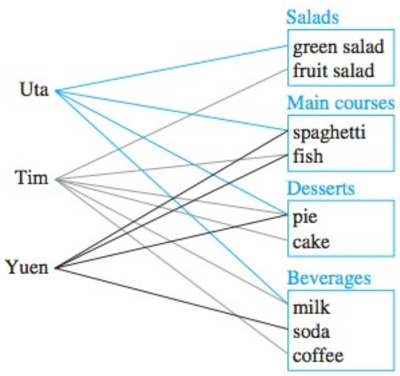
Verbalize and Test Statements



a. **∃ an item *I* such that ∀ students *S*, *S* chose *I*.**

There is an item that was chosen by every student. → true

b. **∃a student *S* such that ∀ items *I*, *S* chose *I*.**

There is a student who chose every available item. →false

c. **∃ a student *S* such that ∀ stations *Z*,∃ an item *I* in *Z* such that *S* chose *I*.**

There is a student who chose at least one item from every station. → true

d. **∀ students *S* and ∀ stations *Z*,∃ an item *I* in *Z* such that *S* chose *I*.**

Every student chose at least one item from every station → false.

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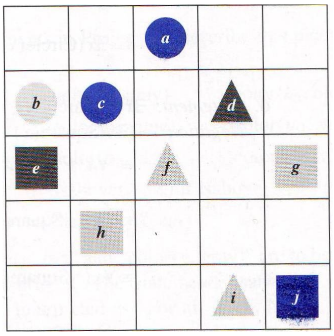
Tarski's world - Formalizing Statements

Describe Tarski's world using universal and external quantifiers
using Formal FOL Notation



a. For all circles x, x is above f.

$\boxed{\forall x(\text{Circle}(x) \to \text{Above}(x, f)).}$

b. There is a square x such that x is black.

$\boxed{\exists x(\text{Square}(x) \wedge \text{Black}(x)).}$

c. For all circles x, there is a square y such that x and y have the same color.

$\boxed{\forall x(\text{Circle}(x) \to \exists y(\text{Square}(y) \wedge \text{SameColor}(x, y))).}$

d. There is a square x such that for all triangles y, x is to right of y.

$\boxed{\exists x(\text{Square}(x) \wedge \forall y(\text{Triangle}(y) \to \text{RightOf}(x, y))).}$

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Formalize these statements

The reciprocal (نظير ضربي) of a real number a is a real number b such that ab = 1. The following two statements are true. Rewrite them formally using quantifiers and variables:

Every nonzero real number has a reciprocal.

$\forall u \in \text{NonZeroR}, \exists v \in \text{R} \;.\; uv = 1.$

There is a real number with no reciprocal.

The number 0 has no reciprocal.

$\exists c \in \mathbf{R} \;\; \forall d \in \mathbf{R} \;\; . \; cd \neq 1.$

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Formalize these statements

There Is a Smallest Positive Integer

$$\exists m \in Z^+ \quad \forall n \in Z^+ \quad . \quad \text{LessOrEqual}(m, n)$$

In the book:

$\exists$ a positive integer m such that $\forall$ positive integers n, $m \leq n$.

There Is No Smallest Positive Real Number

$$\forall x \in R^+ \quad \exists y \in R^+ \quad . \quad \text{Less}(y, x)$$

In the book:

$\forall$ positive real numbers x, $\exists$ a positive real number y such that $y < x$.

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- ❑ Part 2: Formalize/Verbalize Multiple Quantifiers
- ❑ Part 3: **Negations of Multiply-Quantified Statements**
- ❑ Part 4: Example: Using FOL to formalize text (optional)

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Negations of Multiply-Quantified Statements

$\sim(\forall x \text{ in } D, \exists y \text{ in } E \text{ such that } P(x, y)) \equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E, \sim P(x, y).$

$\sim(\exists x \text{ in } D \text{ such that } \forall y \text{ in } E, P(x, y)) \equiv \forall x \text{ in } D, \exists y \text{ in } E \text{ such that } \sim P(x, y).$

Examples:

~ (∀x ∃y . Loves (x,y))

$\exists x \ \forall y . \sim \text{Love}(x,y)$

~(∃x ∀y . Loves (x,y))

$\forall x \ \exists y . \sim \text{Love}(x,y)$

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Negations of Multiply-Quantified Statements

Not all people love someone.

~ (all people love someone)

$\sim(\forall x \ \exists y . \text{Love}(x,y))$

$\exists x \ \forall y . \sim \text{Love}(x,y))$

Some people do not love everyone

Not all people love everyone.

~ (All people love everyone)

$\sim \forall x \forall y \text{ Like}(x, y)$

.....

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Example: Using FOL to formalize text

Example from: Russell & Norvig Book

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

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Example: Using FOL to formalize text

... it is a crime for an American to sell weapons to hostile nations:

$\forall x,y,z \,.\, American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono ... has some missiles, i.e.,

$\exists x \,.\, Owns(Nono,x) \wedge Missile(x)$

... all of its missiles were sold to it by Colonel West

$\forall x \,.\, Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

Missiles are weapons:

$\forall x \,.\, Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile“:

$\forall x \,.\, Enemy(x,America) \Rightarrow Hostile(x)$

West, who is American ...

$American(West)$

The country Nono, an enemy of America ...

$Enemy(Nono,America)$

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